



## Opinion

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## Optimal Innovation and Optimal Imitation: an Integrated Analysis

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## Abstract

Technology creation and technology diffusion are two fundamental pieces of any meaningful assessment of the dynamics of economic growth. Economies grow because they accumulate material inputs, namely physical capital and human capital, but the techniques, procedures and ideas used in production are vital to guarantee efficiency and growth. This opinion paper recovers a recent contribution to the literature on innovation and imitation in order to address and discuss, in an integrated way, the role of innovative activities and technology adoption.

## Introduction

When assessing the role that technology plays in fostering economic growth, it is usual to conceive a technology frontier, where the most research advanced economy is placed, and relatively to which all the other economies, the followers, will want to catch-up. Catching-up is typically feasible through imitation; economies that do not possess conditions to undertake cutting-edge research will want to adopt the technology that the technological leader has already tested and successfully implemented [1].

The study of technology creation and how it diffuses has a long tradition in economics, that goes back to Nelson and Phelps and that has counted with a large number of seminal contributions (among many others, Jovanovic and Rob, Segerstrom, Parente and Prescott, Mukoyama, Peyton-Young,). A recent contribution relating the subject under discussion, which is analyzed in this paper, is the one by Benhabib, Perla and Tonetti, henceforth BPT [2-4].

In BPT a fully deterministic model is designed to address the optimal decisions of the economies on whether to innovate or imitate the leader. A single optimal control problem allows to address both the challenge of the country in the technology frontier and the behavior of all the followers, and it is precisely this ability to integrate innovation and technology adoption under the same simple modeling structure that makes this an appealing framework upon which to think about how knowledge creation and knowledge diffusion take place in the world we live in [5-7].

## The Optimal Control Problem

The dynamic optimization model proposed by BPT assumes a state variable,  $z(t)$ , which represents the aggregate productivity level of the economy. Two control variables are also taken: the expenditures on innovation,  $\gamma(t)$ , and the expenditures on technology adoption,  $s(t)$ . In the model's structure, the following parameters are relevant:  $B > 0$  represents the output of one productivity unit;  $\sigma > 0$  is the benefit obtained by one unit of expenditure on innovation, and  $\rho > 0$  is the intertemporal discount rate. Furthermore, one needs to consider the benefit obtained by one unit of expenditure on adoption, and this is given by the following diffusion function:

$$D(z) = \frac{c}{m} \left[ 1 - \left( \frac{z(t)}{F(t)} \right)^m \right] \quad (1)$$

In Eq. (1),  $F(t)$  represents the technology frontier. Observe that  $z(t) \leq F(t)$  necessarily holds; in the case of the technology leader economy,  $z(t) = F(t)$ . In order to obtain economically meaningful

results, the following constraints on parameter values apply:  $m-1, c < (2+m)\sigma$  (these two conditions correspond to assumption 3 in BPT).

The optimal control problem of the representative agent of an economy that intends to maximize the utility of the net value of productivity takes the form,

$$\max_{s(t), \gamma(t)} Z(0) \equiv \int_0^{+\infty} \left\{ \ln[Bz(t) - s(t)z(t) - \gamma(t)z(t)] \right\} \exp(-\rho t) dt \quad (2)$$

$$\text{subject to: } \frac{\dot{z}(t)}{z(t)} = \sigma\gamma(t) + D(z)s(t), \quad z(0) \text{ given}$$

$$s(t) \geq 0, \gamma(t) \geq 0$$

The optimization problem in Eq. (2) furnishes the following information: the economy desires to maximize the discounted current value of the sum of the utility generated by a given level of productivity, from now to an infinite future horizon. The utility is logarithmic and the argument of the function is the difference between the output obtained for the available productivity level and the costs incurred by the economy both of diffusion and of innovation. This optimization problem is subject to a constraint on the growth of productivity: productivity increases with the investment made both in innovation and in adoption.

## The Economy at the Frontier

Problem (2) is faced by every country in the assumed world economy, and can be solved independently for each of the countries. It simplifies, though, for one economy, namely the economy that stands on the technological frontier and therefore has no catching-up to do. This economy will have no need of investing in adopting the technology of others and, thus,  $s(t) = 0$ . For this economy, by definition,  $z(t) = F(t)$ . Under these two simplifying constraints, problem (2) reduces to:

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$$\max_{s(t), \gamma(t)} Z(0) \equiv \int_0^{+\infty} \left\{ \ln[Bz(t) - \gamma(t)z(t)] \right\} \exp(-\rho t) dt \quad (3)$$

$$\text{subject to: } \frac{\bar{z}(t)}{z(t)} = \sigma\gamma(t), \quad z(0) \text{ given}$$

$$\gamma(t) \geq 0$$

The problem of the leader, i.e., of the economy at the knowledge frontier, in Eq. (3), is straightforward to solve resorting to trivial optimal control techniques. The respective current value Hamiltonian function is,

$$H[z(t), \gamma(t), p(t)] = \ln[Bz(t) - \gamma(t)z(t)] + p(t)\sigma\gamma(t)z(t) \quad (4)$$

with  $p(t)$  the co-state variable associated with  $z(t)$ . The first-order optimality conditions are:

$$\frac{\partial H}{\partial \gamma} = 0 \Rightarrow \sigma[B - \gamma(t)]p(t)z(t) = 0 \quad (5)$$

$$\bar{p}(t) = \rho p(t) - \frac{\partial H}{\partial z} \Rightarrow \bar{p}(t) = [\rho - \sigma\gamma(t)]p(t) - 1/z(t) \quad (6)$$

and the transversality condition

$$\lim_{t \rightarrow +\infty} p(t)z(t)\exp(-\rho t) = 0 \quad (7)$$

To evaluate the dynamics underlying the model, let  $\mu(t) \equiv p(t)z(t)$ . For this new variable,

$$\frac{\bar{\mu}(t)}{\mu(t)} = \frac{\bar{p}(t)}{p(t)} + \frac{\bar{z}(t)}{z(t)} = \rho - 1/\mu(t) \quad (8)$$

Eq. (8) is a one-dimensional differential equation involving a single unstable steady-state point,  $\bar{\mu} = 1/\rho$ . Unless the system rests in this point, the transversality condition (7) is violated and the optimization of (3) does not take place. Hence, the technological leader will select a path for  $\gamma(t)$  such that  $\mu(t)$  remains in the mentioned point, i.e.,  $\mu(t) = 1/\rho$ .

Eq. (5) is equivalent to  $\sigma[B - \gamma(t)] = 1/\mu(t)$ . For the specified value of  $\mu(t)$ , one arrives to a constant value for  $\gamma(t)$ ,

$$\gamma(t) = B - \rho/\sigma \quad (9)$$

Replacing  $\gamma(t)$  as displayed in Eq. (9) into the state constraint of the problem, one concludes that the growth of the productivity level of the technological leader, which is also the growth of the technology frontier, is constant through time,

$$\frac{\bar{z}(t)}{z(t)} = \frac{\bar{F}(t)}{F(t)} = \sigma B - \rho \quad (10)$$

The technological frontier problem provides the information that the country in this position will optimally select a constant over time level of expenditures on innovation, as given by Eq. (9) and, as a result, the productivity frontier will grow at a constant rate over time, namely (10).

## The Followers

After accessing the problem of the leader one can now concentrate on the problem of the followers, that are all the other economies, namely those for which  $z(t) < F(t)$ . Unlike the economy at the frontier, to the followers adoption eventually matters. The problem to be solved

by the followers is the one in (3), with an additional constraint which is that the growth of  $F(t)$  is given by (10). Again, one sets up the current value Hamiltonian function, which is now

$$H[z(t), F(t), \gamma(t), s(t), p(t), q(t)] = \ln[Bz(t) - s(t)z(t) - \gamma(t)z(t)] + p(t)z(t)[\sigma\gamma(t) + D(z)s(t)] + q(t)[\sigma B - \rho]F(t) \quad (11)$$

In Eq. (11),  $p(t)$  and  $q(t)$  are both co-state variables or shadow-prices. Among the optimality conditions, we find the following two, regarding the derivatives of the Hamiltonian with respect to the control variables,

$$\frac{\partial H}{\partial \gamma} = 0 \Rightarrow \sigma[B - s(t) - \gamma(t)]p(t)z(t) = 0 \quad (12)$$

$$\frac{\partial H}{\partial s} = 0 \Rightarrow D(z)[B - s(t) - \gamma(t)]p(t)z(t) = 0 \quad (13)$$

Combining Eqs. (12) and (13), one arrives to the result,

$$\frac{z(t)}{F(t)} = \left(1 - \frac{\sigma m}{c}\right)^{1/m} \quad (14)$$

Eq. (14) indicates that, under conditions of optimality, the ratio between the economy's productivity level and the productivity at the frontier is a constant value. This implies that equality  $\frac{z(t)}{z(t)} = \frac{F(t)}{F(t)}$  would hold for every economy. Each follower would choose, in this circumstance, trajectories for the control variables, concerning expenditures in innovation and adoption, that would satisfy the equality between productivity growth rates; straightforward computation indicates that

$$s(t) + \gamma(t) = B - \rho/\sigma \quad (15)$$

Under Eq. (15), the technology adopter will have to choose a combination between investing in innovation and investing in adoption such that the sum of investments is a constant value. Note that, as stressed by BPT, it will be indifferent to imitate or to innovate as long as condition (15) is met.

## The Diversity of Dynamics

According to BPT, the interesting and appealing results of the model arise when one realizes that initial conditions are likely to prevent result (14) to be verified. In fact, the ratio  $z(t)/F(t)$  is a threshold that will hardly apply to any of the followers. Thus, two cases are possible beyond the knife edge condition:

$$i. \frac{z(0)}{F(0)} < \left(1 - \frac{\sigma m}{c}\right)^{1/m};$$

$$ii. \frac{z(0)}{F(0)} > \left(1 - \frac{\sigma m}{c}\right)^{1/m}.$$

In BPT, a rigorous proof of how the dynamics unfold is presented. Here, we just refer to the most meaningful results. If initial conditions are such that inequality ii. holds, then the followers will optimally choose to be only innovators ( $s=0$ ). The productivity in these economies will grow at an exact same rate as the economy at the frontier. Therefore, no divergence or convergence from each economy relatively to the leader will take place.

For the economies in which condition i. is observed, the optimal

choice is to invest solely in technology adoption ( $\gamma=0$ ). These economies will converge to a long-term productivity ratio  $\bar{z}/\bar{F}$  that is lower than the threshold given by (14). Two possibilities exist in this case: economies for which  $\frac{z(0)}{F(0)} < \bar{z}/\bar{F}$  will catch-up to  $\bar{z}/\bar{F}$ , while in the opposite case,  $\frac{z(0)}{F(0)} > \bar{z}/\bar{F}$  will regress and fall back to  $\bar{z}/\bar{F}$ .

## Epilogue

The BPT model shows how a relatively simple deterministic theoretical structure may assist in explaining relevant stylized facts about how economies grow. Empirical evidence points to a diversity of productivity growth patterns. In the mentioned article, although all economies behave similarly and solve a same dynamic planning problem, they end up by following different growth trajectories. Depending on initial conditions, economies may opt to innovate or adopt existing technology. Initially well-endowed economies will grow at the same rate as the economy at the frontier without ever converging or diverging; economies poorly endowed of technology

in the starting date will also grow at the same rate as the innovators in the long-run, but this occurs after a probably long transient phase where some economies catch-up (those with the initially lowest levels of productivity), while others fall back.

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